

Rapidity equilibration in d + Au and Au + Au systems

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Abstract. In a Relativistic Diffusion Model (RDM), the evolution of net-proton rapidity spectra with $\sqrt{s_{NN}}$ in heavy systems is proposed as an indicator for local equilibration and longitudinal expansion. The broad midrapidity valley recently discovered at RHIC in central Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV suggests rapid local equilibration which is most likely due to deconfinement, and fast longitudinal expansion. Rapidity spectra of produced charged hadrons in d + Au and Au + Au systems at RHIC energies and their centrality dependence are well described in a three-sources RDM. In central collisions, about 19% of the produced particles are in the equilibrated midrapidity region for d + Au.

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1 Introduction

The production and identification of a transient quark-gluon plasma in thermal equilibrium is of basic importance in relativistic heavy-ion physics. In this contribution I propose nonequilibrium statistical methods to investigate analytically the gradual thermalization occurring in the course of stopping and particle production at the highest available energies. The approach is tailored to identify the fraction of net baryons, as well as of produced charged hadrons that attain local thermal equilibrium from their distribution functions in rapidity or pseudorapidity. This yields indirect evidence for the extent and system size dependence of a locally equilibrated parton plasma.

Net-baryon rapidity distributions are sensitive indicators for local equilibration and deconfinement in relativistic heavy-ion collisions [1, 2]. At AGS, SPS and RHIC energies these are clearly nonequilibrium distributions even in central collisions [2], and also the longitudinal distributions of produced particles are not fully thermalized in heavy symmetric [3] as well as in light asymmetric systems [4].

To account for the nonequilibrium behavior of the system, the evolution of net-proton rapidity spectra with incident energy $\sqrt{s_{NN}} = 4.9$ to 200 GeV is first studied analytically in a Relativistic Diffusion Model (RDM, sect. 2), and compared to AGS [5], SPS [6] and RHIC [7] central collision data, sect. 3. In addition to the nonequilibrium statistical evolution as described in the RDM, collective longitudinal expansion is considered through a comparison of the analytical RDM solutions with the data. It turns

out that a three-sources RDM with a small, but sizeable (14%) midrapidity source is required from the RHIC net-proton data.

The midrapidity source is more important for produced charged hadrons. In central d + Au collisions at RHIC energies, almost 20% of the produced particles are in the equilibrium source, sect. 4. Here the centrality dependence is also investigated in detail. The fraction of produced particles in the midrapidity source tends to be larger in central Au + Au collisions [3]. The conclusions are drawn in sect. 5.

2 Relativistic diffusion model

The simplified model ([2], and references therein) as discussed in this contribution is based on a linear Fokker-Planck equation (FPE) for the components k of the distribution function $R(y, t)$ in rapidity space, $y = 0.5 \cdot \ln((E + p)/(E - p))$,

$$\frac{\partial}{\partial t} R_k(y, t) = \frac{1}{\tau_y} \frac{\partial}{\partial y} \left[(y - y_{eq}) \cdot R_k(y, t) \right] + \frac{\partial^2}{\partial^2 y} \left[D_y^k \cdot R_k(y, t) \right]. \quad (1)$$

The diagonal components D_y^k of the diffusion tensor contain the microscopic physics in the respective projectile-like ($k = 1$), target-like ($k = 2$) and central ($k = 3$) regions. They account for the broadening of the distribution functions through interactions and particle creations. In the present investigation the off-diagonal terms of the diffusion tensor are assumed to be zero. The rapidity re-

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laxation time τ_y determines the speed of the statistical equilibration in y -space.

As time goes to infinity, the mean values of the solutions of eqs. (1) approach the equilibrium value y_{eq} . This value is zero for symmetric systems, but for asymmetric systems like d + Au it deviates from zero. We determine it from energy and momentum conservation [8,9] in the system of target- and projectile-like participants and hence, it depends on impact parameter. This dependence is decisive for a detailed description of the measured rapidity distributions in asymmetric systems:

$$y_{eq}(b) = 1/2 \cdot \ln \frac{\langle m_1^T(b) \rangle \exp(y_{max}) + \langle m_2^T(b) \rangle \exp(-y_{max})}{\langle m_2^T(b) \rangle \exp(y_{max}) + \langle m_1^T(b) \rangle \exp(-y_{max})} \quad (2)$$

with the beam rapidities $y_b = \pm y_{max}$, the transverse masses $\langle m_{1,2}^T(b) \rangle = \sqrt{m_{1,2}^2(b) + \langle p_T \rangle^2}$, and masses $m_{1,2}(b)$ of the respective participants that depend on the impact parameter b . The average numbers of participants $N_{1,2}(b)$ in the incident nuclei are calculated from the geometrical overlap, or from Glauber theory.

The RDM describes the drift towards y_{eq} in a statistical sense. Whether the mean values of the distribution functions R_1 and R_2 actually attain y_{eq} depends on the interaction time τ_{int} (the time the system interacts strongly, or the integration time of (1)). It can be determined from dynamical models or from parametrizations of two-particle correlation measurements. For central Au + Au at 200 A GeV, this yields about $\tau_{int} \simeq 10$ fm/c, which is too short for R_1 and R_2 in order to reach equilibrium. Note, however, that this does not apply to R_3 which is born near local equilibrium at short times, and then spreads in time through interactions with other particles at nearly the same rapidity.

The analytical diffusion model is consistent with, and complementary to parton cascade models where stopping involves large sudden jumps in rapidity from hard scatterings (*e.g.* [10]), because even hard partons can participate significantly in equilibration processes, as is evidenced by the high- p_T suppression found in Au + Au at RHIC.

Nonlinear effects are not considered here. Their possible role in the context of relativistic heavy-ion collisions has been discussed elsewhere ([2], and references therein). These account to some extent for the collective expansion of the system in y -space, which is not included *a priori* in a statistical treatment. In the linear model, the expansion is treated through effective diffusion coefficients D_y^{eff} that are larger than the theoretical values calculated from the dissipation-fluctuation theorem that normally relates D_y and τ_y to each other [1]. This relation has been derived from the requirement that the stationary solution of eq. (1) is equated with a Gaussian approximation to the thermal equilibrium distribution in the y -space. At fixed incident energy, this weak-coupling rapidity diffusion coefficient turns out to be proportional to the equilibrium temperature T as in the analysis of Brownian motion (Einstein relation)

$$D_y \propto \frac{T}{\tau_y}. \quad (3)$$

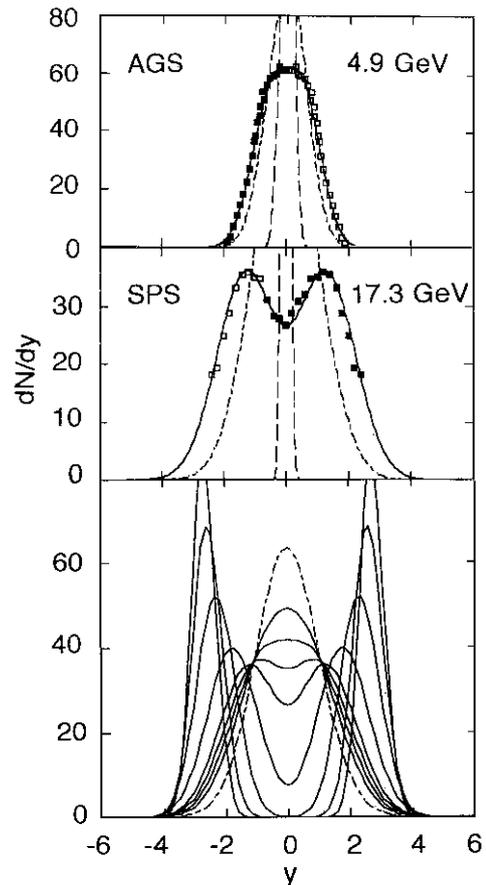


Fig. 1. Net-proton rapidity spectra in the two-sources Relativistic Diffusion Model (RDM), solid curves, compared to central Au + Au data from AGS, Pb + Pb from SPS (middle), and RDM solutions for various t/τ_y (bottom). From [11].

One then deduces the collective expansion velocities from a comparison between data (yielding D_y^{eff}) and theoretical result (D_y), as described in more detail in [11].

The FPE can be solved analytically in the linear case with constant D_y^k . For net-baryon rapidity distributions, the initial conditions are δ -functions at the beam rapidities $y_b = \pm y_{max}$. However, it has been shown that in addition there exists a central ($k = 3$, equilibrium) source at RHIC energies which accounts for about 14% of the net-proton yield in Au + Au collisions at 200 A GeV [1], and is most likely related to deconfinement. For produced particles the central source turns out to be even more important.

3 Net-proton rapidity distributions

Comparing the solutions of eq. (1) to Au + Au central collision (5 per cent of the cross-section) data at AGS energies $\sqrt{s_{NN}} = 4.9$ GeV [5], it turns out that due to the relatively long interaction time τ_{int} and hence, the large ratio $\tau_{int}/\tau_y \simeq 1.08$, the system is in rapidity space very close to thermal equilibrium (dash-dotted curve), with longitudinal collective expansion at $v_{coll}^{\parallel} = 0.49$ (fig. 1, upper

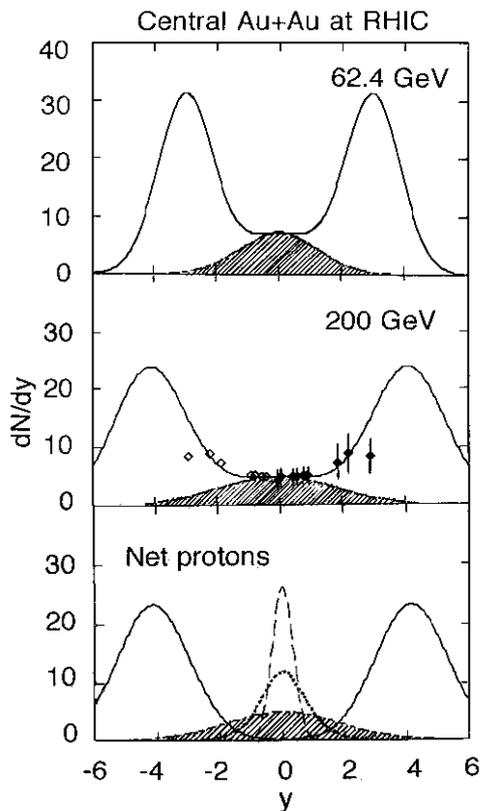


Fig. 2. Net-proton rapidity spectra in the three-sources Relativistic Diffusion Model (RDM), solid curves, compared to central Au + Au data from RHIC. From [11].

frame) [2]. The bell-shaped experimental distribution is in good agreement with the solution of eq. (1). The distributions remain bell-shaped also at lower energies [5]. This situation changes at the higher SPS energy of $\sqrt{s_{NN}} = 17.3$ GeV (fig. 1, middle). Here net-proton Pb + Pb rapidity spectra corrected for hyperon feeddown [6] show two pronounced peaks in central collisions, which arise from the penetration of the incident baryons through the system. The gradual slow-down and broadening is described using eq. (1) as a hadronic diffusion process with subsequent collective expansion, $v_{coll}^{\parallel} = 0.75$. The associated nonequilibrium solutions with expansion for various values of τ_{int}/τ_y are shown in the lower frame of fig. 1. The system clearly does *not* reach the dash-dotted equilibrium solution. Hence, both nonequilibrium properties, *and* collective expansion are required to interpret the broad rapidity spectra seen at the SPS.

Within the current framework, no indication for deconfinement of the incident baryons or other unusual processes can be deduced from the net-proton rapidity data at AGS and SPS energies, because the crucial midrapidity region is here too small.

This is, however, different at RHIC energies $\sqrt{s_{NN}} = 200$ GeV. The RDM nonequilibrium solution exhibits pronounced penetration peaks with collective longitudinal expansion $v_{coll}^{\parallel} = 0.93$, but it fails to reproduce the

BRAHMS net-proton data [7] in the broad midrapidity valley (solid curve in fig. 2, bottom): The diffusion of the incident baryons due to soft scatterings is not strong enough to explain the net-baryon density in the central rapidity region. The individual nonequilibrium solutions R_1 and R_2 are Gaussians, and if they fit the data points near $y = \pm 2$, they necessarily grossly underpredict the midrapidity yield because it is in their tails.

This central region can only be reached if a fraction of the system undergoes a fast transition to local thermal equilibrium, dashed curve in fig. 2, bottom. With collective expansion of this locally equilibrated subsystem of 22 net protons ($v_{coll}^{\parallel} = 0.93$, dashed areas), the flat midrapidity BRAHMS data are well reproduced in an incoherent superposition of nonequilibrium and equilibrium solutions of (1)

$$\frac{dN(y, t = \tau_{int})}{dy} = N_1 R_1(y, \tau_{int}) + N_2 R_2(y, \tau_{int}) + N_{eq} R_{eq}^{loc}(y) \quad (4)$$

with the interaction time τ_{int} (total integration time of the differential equation). The fast transition of a subsystem of $N_{eq} \simeq 55$ baryons, or 22 protons to local thermal equilibrium implies that in this region of rapidity space, baryon diffusion has been replaced by hard scatterings of the associated participant partons with large rapidity transfer, and subsequent thermal equilibration.

In a schematic calculation for the lower RHIC energy of $\sqrt{s_{NN}} = 62.4$ GeV, I have used the rapidity diffusion coefficient from 200 GeV to obtain the result in the upper frame of fig. 2. Two pronounced penetration peaks can be seen, together with a narrow midrapidity valley. The corresponding data have been taken, and are presently being analyzed by the BRAHMS Collaboration. Once they are available, an adjustment of the effective diffusion coefficient and N_{eq} may be required.

4 Particle production at RHIC

Whereas local equilibration in rapidity space occurs only for a small fraction of the participant baryons in central collisions at RHIC energies, the equilibrium fraction tends to be larger for produced hadrons. This can be inferred from recent applications of the RDM with three sources (located at the beam rapidities, and at the equilibrium value) and δ -function initial conditions [3, 4]. After conversion to pseudorapidity space, the calculations by Biyajima *et al.* for heavy systems [3] within the three-sources RDM yield good agreement with Au + Au data at both 130 and 200 GeV per particle pair. As an example, the result for 130 GeV is shown in fig. 3 in comparison with PHOBOS data; the fit at 200 GeV is of similar quality. Recently, pseudorapidity distributions of primary charged particles have become available [12] as functions of centrality in d + Au collisions at a nucleon-nucleon center-of-mass energy of 200 GeV. They are also investigated here within the 3-sources RDM framework.

For produced particles, the initial conditions are not uniquely defined. The previous experience with the

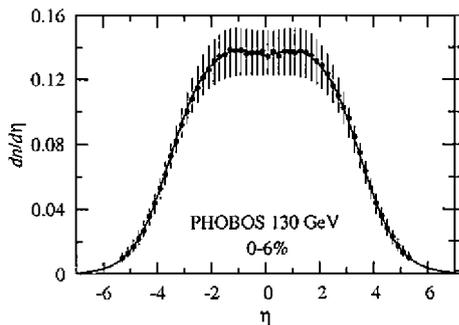


Fig. 3. Charged-hadron rapidity spectrum in the three-sources Relativistic Diffusion Model (RDM), solid curve, compared to central Au + Au data from RHIC/PHOBOS. From Biyajima *et al.* [3].

Au + Au system regarding both net baryons [1], and produced hadrons [3] favors a three-sources approach, with δ -function initial conditions at the beam rapidities, supplemented by a source centered at the equilibrium value y_{eq} .

Physically, the particles in this source are expected to be generated mostly from gluon-gluon collisions since only few valence quarks are present in the midrapidity region at $\sqrt{s_{NN}} = 200$ GeV [1]. The final width of this source corresponds to the local equilibrium temperature of the system which may approximately be obtained from analyses of particle abundance ratios, plus the broadening due to the collective expansion of the system. Formally, the local equilibrium distribution is a solution of (1) with diffusion coefficient $D_y^3 = D_y^{eq}$, and δ -function initial condition at the equilibrium value.

The PHOBOS Collaboration has analyzed their minimum-bias data successfully using a triple-Gaussian fit [13]. This is consistent with our analytical three-sources approach, although additional contributions to particle production have been proposed. Beyond the precise representation of the data, however, the relativistic diffusion model offers an analytical description of the statistical equilibration during the collision and in particular, of the extent of the moving midrapidity source which is indicative of a locally equilibrated parton plasma prior to hadronization.

With δ -function initial conditions for the Au-like source (1), the d-like source (2) and the equilibrium source (eq), I obtain exact analytical diffusion model solutions as an incoherent superposition of the distribution functions $R_k(y, t)$. The three individual distributions are Gaussians with mean values

$$\langle y_{1,2}(t) \rangle = y_{eq}[1 - \exp(-t/\tau_y)] \mp y_{max} \exp(-t/\tau_y) \quad (5)$$

for the sources 1 and 2, and y_{eq} for the moving equilibrium source. Hence, all three mean values attain $y_{eq}(b)$ as determined from (2) for $t \rightarrow \infty$, whereas for short times the mean rapidities are smaller than, but close to the Au- and d-like (absolute) values in the sources 1 and 2. The variances are

$$\sigma_{1,2,eq}^2(t) = D_y^{1,2,eq} \tau_y [1 - \exp(-2t/\tau_y)]. \quad (6)$$

The charged-particle distribution in rapidity space is then obtained as incoherent superposition of nonequilibrium and local equilibrium solutions of (1) as in (4), with N_k replaced by N_{ch}^k . The average numbers of charged particles in the Au- and d-like regions $N_{ch}^{1,2}$ are proportional to the respective numbers of participants $N_{1,2}$,

$$N_{ch}^{1,2} = N_{1,2} \frac{(N_{ch}^{tot} - N_{ch}^{eq})}{(N_1 + N_2)} \quad (7)$$

with the constraint $N_{ch}^{tot} = N_{ch}^1 + N_{ch}^2 + N_{ch}^{eq}$. Here the total number of charged particles in each centrality bin N_{ch}^{tot} is determined from the data. The average number of charged particles in the equilibrium source N_{ch}^{eq} is a free parameter that is optimized together with the variances and τ_{int}/τ_y in a χ^2 -fit of the data using the CERN minuit code.

The results for $N_{1,2}$ in a geometrical overlap calculation are consistent with the Glauber calculations reported in [12] which we use in the further analysis. The corresponding equilibrium values of the rapidity vary from $y_{eq} = -0.169$ for peripheral (80–100%) to $y_{eq} = -0.944$ for central (0–20%) collisions. They are negative due to the net longitudinal momentum of the participants in the laboratory frame, and their absolute magnitudes decrease with impact parameter since the number of participants decreases for more peripheral collisions.

The result of the RDM calculation is shown in fig. 4 (right column, middle) for central collisions (0–20%) of d + Au. The charged-particle yield is dominated here by hadrons produced from the Au-like source, but there is a sizeable equilibrium source (dash-dotted line) that is more important than the d-like contribution. This thermalized source is moving since y_{eq} has a finite negative value for d + Au, whereas it is at rest for symmetric systems.

The total yield is compared to PHOBOS data [12] which refer to the pseudorapidity $\eta = -\ln[\tan(\theta/2)]$ since particle identification was not available. As a consequence, there is a small difference to the model result [4] in y -space ($y \approx \eta$) which is most pronounced in the midrapidity region. It is removed when the theoretical result is converted to η -space through the Jacobian

$$J(\eta, \langle m \rangle / \langle p_T \rangle) = \cosh(\eta) \cdot [1 + (\langle m \rangle / \langle p_T \rangle)^2 + \sinh^2(\eta)]^{-1/2}. \quad (8)$$

Here the average mass $\langle m \rangle$ of produced charged hadrons in the central region is approximated by the pion mass m_π , and a mean transverse momentum $\langle p_T \rangle = 0.4$ GeV/ c is used. In the Au-like region, the average mass is larger due to the participant protons, but since their number is small compared to the number of produced charged hadrons in the d + Au system, the increase above the pion mass remains small, and it turns out to have a negligible effect on the results.

The magnitude of the equilibrium source for particle production is about 19% in the light and asymmetric d + Au system, whereas it tends to be larger in the Au + Au system [3].

The model calculations are compared with PHOBOS data for five centrality cuts [12] and minimum bias [13] in

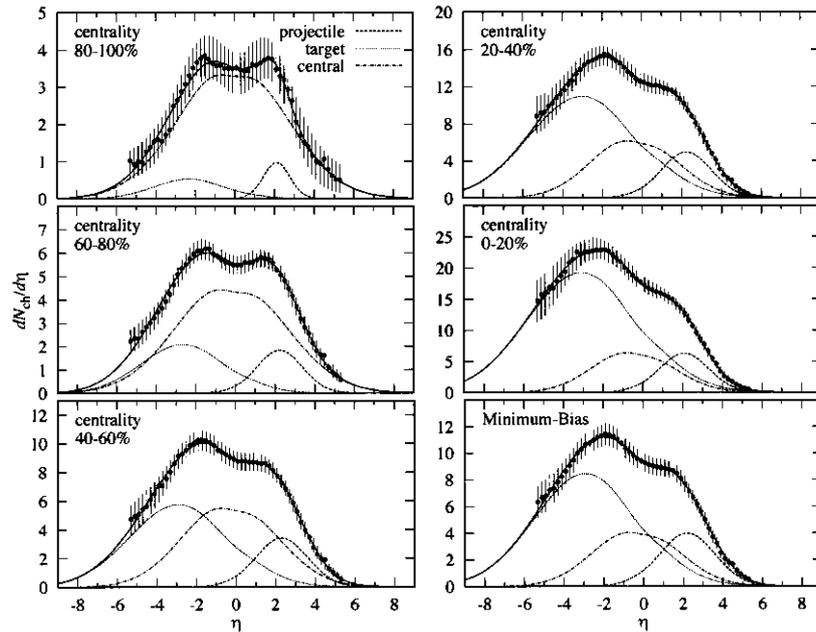


Fig. 4. Charged-hadron rapidity spectra in the three-sources Relativistic Diffusion Model (RDM), solid curves, compared to d + Au data from RHIC/PHOBOS for various centralities. From [14].

fig. 4. The observed shift of the distributions towards the Au-like region in more central collisions, and the steeper slope in the deuteron direction as compared to the gold direction appear in the relativistic diffusion model as a consequence of the gradual approach to equilibrium: The system is on its way to statistical equilibrium, but it does not fully reach it.

Given the structure of the differential equation that is used to model the equilibration, together with the initial conditions and the constraints imposed by eqs. (2) and (7), there is no room for substantial modifications of this result. In particular, changes in the impact parameter dependence of the mean values in eq. (5) that are not in accordance with eq. (2) vitiate the precise agreement with the data.

5 Conclusion

To conclude, I have interpreted recent results for central collisions of heavy systems at AGS, SPS and RHIC energies in a Relativistic Diffusion Model (RDM) for multiparticle interactions based on the interplay of nonequilibrium and local equilibrium (“thermal”) solutions. In the linear version of the model, analytical results for the rapidity distribution of net protons in central collisions have been obtained and compared to data.

The enhancement of the diffusion in rapidity space as opposed to the expectation from the weak-coupling dissipation-fluctuation theorem has been interpreted as collective expansion, and longitudinal expansion velocities for net protons have been determined from a comparison between RDM results and data based on a relativistic expression for the collective velocity, cf. [11] for details. A prediction for net-proton rapidity distributions

at $\sqrt{s_{NN}} = 62.4$ GeV yields a smaller midrapidity valley than at 200 GeV which will soon be compared with forthcoming data.

Charged-particle production in d + Au collisions at $\sqrt{s_{NN}} = 200$ GeV as a function of centrality has also been investigated within the framework of my analytically soluble three-sources model, and compared to Au + Au. Excellent agreement with recent PHOBOS pseudorapidity distributions has been obtained. For central collisions, a fraction of only 19% of the produced particles arises from the locally equilibrated midrapidity source, whereas this figure tends to be larger in Au + Au collisions at the same energy.

At $\sqrt{s_{NN}} = 200$ GeV, the midrapidity source for particle production must essentially have partonic origin, because the net-proton rapidity spectra for Au + Au show that the nucleon (and hence, valence-quark) content in this region of rapidity space is small [1]. Since the source is in local thermal equilibrium, it is likely that the generating partons form a thermalized plasma prior to hadronization.

The d + Au results show clearly that only the midrapidity part of the distribution function comes very close to thermal equilibrium, whereas the interaction time is too short for the d- and Au-like parts to attain the thermal limit. The same is true for the heavy Au + Au system at the same energy, but there the precise fraction of particles produced in the equilibrium source is more difficult to determine due to the symmetry of the problem.

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